Phenomenology of effective gravity

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Abstract

The cosmological constant is not an absolute constant. The gravitating part of the vacuum energy is adjusted to the energy density of matter and to other types of the perturbations of the vacuum. We discuss how the vacuum energy responds (i) to the curvature of space in the Einstein closed Universe; (ii) to the expansion rate in the de Sitter Universe; and (iii) to the rotation in the Gödel Universe. In all these steady state Universes, the gravitating vacuum energy is zero in the absence of the perturbation, and is proportional to the energy density of perturbation. This is in a full agreement with the thermodynamic Gibbs-Duhem relation applicable to any quantum vacuum. It demonstrates that (i) the cosmological constant is not huge, since according to the Gibbs-Duhem relation the contribution of zero point fluctuations to the vacuum energy is cancelled by the trans-Planckian degrees of freedom; (ii) the cosmological constant is non-zero, since the perturbations of the vacuum state induce the non-zero vacuum energy; and (iii) the gravitating vacuum energy is on the order of the energy density of matter and/or of other perturbations. We also consider the vacuum response to the non-steady-state perturbations. In this case the Einstein equations are modified to include the non-covariant corrections, which are responsible for the relaxation of the cosmological constant. The connection to the quintessence is demonstrated. The

problem of the energy-momentum tensor for the gravitational field is discussed in terms of effective gravity. The difference between the momentum and pseudomomentum of gravitational waves in general relativity is similar to that for sound waves in hydrodynamics.

1 Introduction

Observations demonstrate that the pressure of the vacuum in our Universe is very close to zero as compared to the natural value which follows from the Planck energy scale, $P \ll P_{\rm Planck}$. The natural value $P_{\rm Planck} \sim \pm E^4_{\rm Planck}$ for the vacuum pressure and vacuum energy, obtained by summing the zero point energies of quantum fields, is by about 120 orders of magnitude exceeds the experimental limit. This is the main cosmological constant problem [1, 2, 3, 4].

Exactly the same 'paradox' occurs in any quantum liquid (or in any other condensed matter). The experimental energy of the ground state of, say, the quantum liquid at $P = 0$ is zero. On the other hand, if one starts calculating the vacuum energy summing the energies of all the positive and negative energy modes up to the natural cut-off energy scale, one obtains the huge energy on the order of $E^4_{\rm Planck}$, where the role of the Planck energy scale is played by the Debye temperature. However, there is no real paradox in quantum liquids, since if in addition to the sub-Planckian modes one adds the trans-Planckian (microscopic, atomic) modes one immediately obtains the zero value, irrespective of the details of the microscopic physics [5]. Thus the fully microscopic consideration restores the Gibbs-Duhem relation, $\rho = -P$, between the energy (the relevant thermodynamic potential) and the pressure of the quantum liquid at $T = 0$. This Gibbs-Duhem relation ensures the nullification of the energy of the vacuum state, $\rho = 0$, if the external pressure is zero.

This is the main message from condensed matter to the physics of the quantum vacuum: One should not worry about the huge vacuum energy, the trans-Planckian physics with its degrees of freedom will do all the job of the cancellation of the vacuum energy without any fine tuning and irrespective of the details of the trans-Planckian physics.

There are other messages which are also rather general and do not depend much on details of the trans-Planckian physics. For example, the gravitating

energy of the perturbed vacuum is non-zero, and it is proportional to the energies related to perturbations. Thus, if the Planck ether (the vacuum) also obeys the Gibbs-Duhem relation, then the cosmological constant Λ is not a constant but is an evolving physical parameter, and our goal is to find its response to different perturbations of the quantum vacuum. We first consider the response of Λ to the steady state perturbations, such as the spatial curvature, steady state expansion and rotation. The response can be obtained either using the Einstein equations or in a pure phenomenological way. Then we consider the response to the time-dependent perturbations of the vacuum. For that we need some modification of the Einstein equations to allow Λ to vary in time, and we shall discuss the phenomenology of this modification. Ref. [6] represents the short version of this paper.

2 Gravity as perturbation of quantum vacuum

2.1 Einstein theory in standard formulation

Let us start with the non-dissipative equations for gravity – the Einstein equations. They are obtained from the action:

$$S = S_{\rm E} + S_\Lambda + S^{\rm M} \tag{1}$$

where $S^{\rm M}$ is the matter action,

$$S_{\rm E} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\mathcal{R} \tag{2}$$

is the Einstein curvature action, and

$$S_\Lambda = -\frac{\Lambda}{8\pi G}\int d^4x\sqrt{-g}\ , \tag{3}$$

where Λ is the cosmological constant introduced by Einstein in 1917 [7], which now got some experimental evidences. The variation over the metric $g^{\mu\nu}$ gives the Einstein equations

$$2\delta S = \delta g^{\mu\nu}\sqrt{-g}\left[-\frac{1}{8\pi G}\left(R_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu}\right) + \frac{\Lambda}{8\pi G}g_{\mu\nu} + T^{\rm M}_{\mu\nu}\right] = 0\ , \tag{4}$$

where $T^{\rm M}_{\mu\nu}$ is the energy-momentum tensor for matter. The Eq.(4) has been originally written in the form where the matter term is on the rhs of the equation, while the gravitational terms which contain two constants G and Λ are on the lhs (see the Einstein paper [7]):

$$\frac{1}{8\pi G}\left(G_{\mu\nu} - \Lambda g_{\mu\nu}\right) = T^{\rm M}_{\mu\nu} \ , \tag{5}$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\mathcal{R} g_{\mu\nu} \tag{6}$$

is the Einstein tensor. The form (5) of the Einstein equation implies that the matter fields serve as the source of the gravitational field.

2.2 Cosmological constant as vacuum energy

Later the cosmological constant term was moved to the rhs of the Einstein equation:

$$\frac{1}{8\pi G} G_{\mu\nu} = T^{\rm M}_{\mu\nu} + T^{\rm vac}_{\mu\nu} \ , \tag{7}$$

where in addition to the matter it became another source of the gravitational field and got the meaning of the energy-momentum tensor of the vacuum [8]:

$$T^{\rm vac}_{\mu\nu} = \rho^{\rm vac} g_{\mu\nu} \quad , \quad \rho^{\rm vac} = \frac{\Lambda}{8\pi G} \ , \tag{8}$$

with $\rho^{\rm vac}$ being the vacuum energy density and $P^{\rm vac} = -\rho^{\rm vac}$ being the vacuum pressure.

2.3 Sakharov gravity as elasticity of quantum vacuum

In the induced gravity introduced by Sakharov [9], the gravity is the elasticity of the vacuum. If it is the fermionic vacuum, the effective action for the gravitational field is induced by the vacuum fluctuations of the fermionic matter fields. Such kind of the effective gravity emerges in the fermionic quantum liquids of some universality class [5]. In the induced gravity the Einstein tensor must be also moved to the matter side, i.e. to the rhs, and the Einstein equations acquire the form

$$0 = T^{\rm M}_{\mu\nu} + T^{\rm vac}_{\mu\nu} + T^{\rm gr}_{\mu\nu} \ . \tag{9}$$

Here the tensor

$$T^{\rm gr}_{\mu\nu} = -\frac{1}{8\pi G} G_{\mu\nu} \tag{10}$$

has the meaning of the stress-energy tensor produced by deformations of the (fermionic) vacuum. It describes such elastic deformations of the vacuum, which distort the effective metric field $g_{\mu\nu}$ and thus play the role of the gravitational field. As distinct from the $T^{\rm vac}_{\mu\nu}$ term which is of the 0-th order in gradients of the metric, the $T^{\rm gr}_{\mu\nu}$ term is of the 2-nd order in gradients of $g_{\mu\nu}$. The higher-order gradient terms also naturally appear in induced gravity.

2.4 Conservation of energy and momentum

In the induced gravity the free gravitational field is absent, since there is no gravity in the absence of the quantum vacuum (in the same manner as there is no sound waves in the absense of the quantum liquid). Thus the total energy-momentum tensor comes from the original (bare) fermionic degrees of freedom. That is why all the three contributions to the energy-momentum tensor are obtained by the variation of the total fermionic action over $g^{\mu\nu}$:

$$T^{\rm total}_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = -\frac{1}{8\pi G} G_{\mu\nu} + \rho^{\rm vac} g_{\mu\nu} + T^{\rm M}_{\mu\nu} \ . \tag{11}$$

According to the variational principle, $\delta S/\delta g^{\mu\nu} = 0$, the total energy-momentum tensor is zero, which gives rise to the Einstein equation in the form of Eq.(9).

In the conventional description of general relativity, there is a problem related to the energy-momentum tensor for gravity: it is impossible to combine the general covariance with the conservation of energy and momentum. Attempts to introduce the energy-momentum tensor for the gravitational field led to various non-covariant objects called pseudotensors (see e.g. [10] and references therein). In the induced gravity description, there is no contradiction between the covariance and the conservation laws: each of three components (matter, gravity, and vacuum) obey the covariant conservation law:

$$T^{\mu\ {\rm M}}_{\nu;\mu} = 0 \ \ , \ \ T^{\mu\ {\rm gr}}_{\nu;\mu} = 0 \ \ , \ \ T^{\mu\ {\rm vac}}_{\nu;\mu} = 0 \ , \tag{12}$$

while the total system obeys both the covariant conservation law and the true conservation law:

$$\partial_\mu T^{\mu\ \mathrm{total}}_{\nu} = 0\ . \tag{13}$$

In general relativity, this equation is the consequence of the fact that $T^{\mu\ \mathrm{total}}_{\nu} = 0$.

Actually, in induced gravity, the equation $T^{\mu\ \mathrm{total}}_{\nu} = 0$ is valid only in the lowest orders of the gradient expansion and can be violated by the higher-order terms which come from the Planck-scale physics and do not respect the general covariance. However, the true conservation law, $\partial_\mu(T^{\mu\ \mathrm{covariant}}_{\nu} + T^{\mu\ \mathrm{noncovariant}}_{\nu}) = 0$, must be obeyed since it exists on the microscopic level. This difference between the induced and fundamental gravity can be used for the construction of the post-Einstein equations describing the evolution of the induced cosmological constant.

2.5 Three components of 'cosmic fluid'

In induced gravity there is no much difference between three contributions to the energy-momentum tensor: all three components of the 'cosmic fluid' (vacuum, gravitational field, and matter) come from the original bare fermions. However, in the low-energy corner, where the gradient expansion for the effective action works, one can distinguish between these contributions: (i) some part of the energy-momentum tensor ($T^{\mathrm{M}}_{\mu\nu}$) comes from the excited fermions – quasiparticles – which in the effective theory form the matter. The other parts come from the fermions forming the vacuum – the Dirac sea. The contribution from the vacuum fermionscan be expanded in terms of the gradients of $g_{\mu\nu}$-field. (ii) The zeroth-order term represents the energy-momentum tensor of the homogeneous vacuum – the Λ-term. Of course, the whole Dirac sea cannot be sensitive to the change of the effective infrared fields $g_{\mu\nu}$: only small infrared perturbations of the vacuum, which we are interested in, are described by these effective fields. (iii) The higher-order terms describe the inhomogeneous distortions of the vacuum state, which plays the role of gravity. The second-order term in gradients, the stress tensor $T^{\mathrm{gr}}_{\mu\nu}$, represents the curvature term in the Einstein equations.

2.6 Induced cosmological constant

In the traditional approach the cosmological constant is fixed, and it serves as the source for the metric field: in other words the input in the Einstein equations is the cosmological constant, the output is de Sitter expansion, if matter is absent. In effective gravity, where the gravitational field, the matter fields, and the cosmological 'constant' emerge simultaneously in the low-energy corner, one cannot say that one of these fields is primary and serves as a source for the other fields thus governing their behavor. The cosmological constant, as one of the players, adjusts to the evolving matter and gravity in a self-regulating way. In particular, in the absence of matter ($T^{\rm M}_{\mu\nu} = 0$) the non-distorted vacuum ($T^{\rm gr}_{\mu\nu} = 0$) acquires zero cosmological constant, since according to the 'gravi-neutrality' condition Eq.(9) it follows from equations $T^{\rm M}_{\mu\nu} = 0$ and $T^{\rm gr}_{\mu\nu} = 0$ that $T^{\rm vac}_{\mu\nu} = 0$. In this approach, the input is the vacuum configuration (in a given example there is no matter, and the vacuum is homogeneous), the output is the vacuum energy. In contrast to the traditional approach, here the gravitational field and matter serve as a source of the induced cosmological constant.

2.7 Gibbs-Duhem relation and cosmological constant

This conclusion is supported by the effective gravity and effective QED which emerge in quantum liquids or any other condensed matter system of the special universality class [5]. The nullification of the vacuum energy in the equilibrium homogeneous vacuum state of the system is general and does not depend on the microscopic atomic (trans-Planckian) structure of the system. It follows from the variational principle, or more generally from the Gibbs-Duhem relation applied to the equilibrium vacuum state of the system if it is isolated from the environment.

The Gibbs-Duhem equation for the liquid containing N identical particles relates the energy E of the liquid and its pressure P in equilibrium:

$$E = TS + \mu N - PV \ , \tag{14}$$

where T, S, μ and V are correspondingly the temperature, entropy, chemical potential and volume of the liquid. Let us consider the ground state (the quantum vacuum) of the system, i.e. the state without quasiparticles or other excitations of the quantum vacuum. This ground state also represents

the quantum vacuum for the effective quantum field theory of the interacting Fermi and Bose quasiparticles, which emerges in the liquid at low energy. The relevant Hamiltonian, whose gradient expansion gives rise to the action for the effective quantum field theory and effective gravity, is $\mathcal{H}-\mu\mathcal{N}$ [11]. Thus the energy of the quantum vacuum is

$$E^{\rm vac} = E - \mu N = \langle {\rm vac}|\mathcal{H} - \mu\mathcal{N}|{\rm vac}\rangle ~. \tag{15}$$

Then from the Gibbs-Duhem relation (14) at $T=0$ one obtains the equation of state for the equilibrium quantum vacuum:

$$\rho^{\rm vac} \equiv \frac{E^{\rm vac}}{V} = -P^{\rm vac} ~, \tag{16}$$

which is the same as for the quantum vacuum in general relativity. For the isolated homogeneous vacuum state, the external pressure is absent and one obtains the nullification of the vacuum energy

$$\rho^{\rm vac} = -P^{\rm vac} = 0 ~. \tag{17}$$

This means that in the effective gravity emerging in quantum liquid one has $T^{\rm vac}_{\mu\nu} = 0$, if the vacuum is in complete equilibrium. This is valid for any quantum liquid. If the liquid contains several different species a of atoms, fermionic or bosonic, the corresponding Gibbs-Duhem relation and the relevant vacuum energy are

$$E = TS - PV + \sum_a \mu_a N_a ~,~ \rho^{\rm vac} = \frac{1}{V}\langle {\rm vac}|\mathcal{H} - \sum_a \mu_a\mathcal{N}_a|{\rm vac}\rangle ~. \tag{18}$$

They again lead at $T=0$ to the equation of state (16) for the vacuum, and to the nullification of the vacuum energy of an isolated system.

Since these thermodynamic arguments are valid for any quantum liquid, one may expect that they are valid for the Planck ether too, irrespective of the details of its structure. If so, then extending this general rule to the quantum vacuum of our Universe, one obtains that the non-perturbed vacuum is not gravitating, i.e. in the quiescent flat Universe without matter one has $\Lambda = 0$ without any fine tuning. This means that the contributions to the vacuum energy from the sub-Planckian and trans-Planckian degrees of freedom exactly cancel each other due to the thermodynamic identity, while each contribution is huge, being of the fourth order of the Planck energy scale:

$$\rho^{\rm vac}_{\rm sub} \sim \rho^{\rm vac}_{\rm trans} \sim \pm E^4_{\rm Planck} \quad , \quad \rho^{\rm vac}_{\rm sub} + \rho^{\rm vac}_{\rm trans} = 0 ~. \tag{19}$$

2.8 Cosmological constant from vacuum perturbations

The equation (17) is valid only at zero temperature. If $T \neq 0$, the same Gibbs-Duhem relation demonstrates, that the vacuum energy is proportional to the thermal energy of the quantum liquid. In the case of massless quasiparticles one obtains that $\rho^{\rm vac} \sim T^4$, i.e. the induced vacuum energy density is proportional to the energy density of matter (let us recall, that in quantum liquids the role of matter is played by the quasiparticle excitations).

Now let us consider how this occurs in general relativity, i.e. how the vacuum energy (the cosmological constant) responds to the perturbations of the vacuum, caused by matter, expansion, spatial curvature and rotation. The Einstein equation does not allow us to obtain the time dependence of the cosmological constant, because of the Bianchi identities $G^{\nu}_{\mu;\nu} = 0$ and covariant conservation law for matter fields (quasiparticles) $T^{\nu {\rm M}}_{\mu;\nu} = 0$ which together lead to $\partial_\mu \Lambda = 0$. But the general relativity allows us to obtain the values of the cosmological constant in different stationary or steady state Universes, such as the Einstein static closed Universe [7], the de Sitter expanding Universe [12], and the Göde lrotating Universe [13]. In other words, we shall study the response of the cosmological constant to curvature, expansion and rotation.

We discuss how the values of the cosmological constant can be obtained without solving the Einstein equations. This can be done in a purely phenomenological way, in which the equilibrium conditions and equations of state for the three components of the system (vacuum, gravitational field and matter) are used.

3 Robertson-Walker metric and its energy momentum tensor

3.1 Einstein action

Let us start with the Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \ . \tag{20}$$

Its Ricci tensor R^μ_ν and Ricci scalar $\mathcal{R}$ are

$$R^0_0 = -3\frac{\partial^2_t a}{a}\ ,\ R^i_j = -\delta^i_j\left(\frac{\partial^2_t a}{a} + 2H^2 + \frac{2k}{a^2}\right)\ ,\ \mathcal{R} = -6\left(\frac{\partial^2_t a}{a} + H^2 + \frac{k}{a^2}\right)\ , \tag{21}$$

where $H = \partial_t a/a$ is the time-dependent Hubble's parameter. After integration by parts the action for the gravitational field becomes quadratic in $\partial_t a$:

$$S^{\rm gr} = -\frac{1}{16\pi G}\int\sqrt{-g}\mathcal{R} = -\frac{1}{16\pi G}\int\sqrt{-g}\tilde{\mathcal{R}} \equiv -\frac{6}{16\pi G}\int a^3\left(H^2 - \frac{2k}{a^2}\right)\ . \tag{22}$$

The Einstein tensor $G^\mu_\nu = R^\mu_\nu - (1/2)\mathcal{R}\delta^\mu_\nu$ is

$$G^0_0 = 3\left(H^2 + \frac{k}{a^2}\right)\ ,\ G^i_j = \delta^i_j\left(2\frac{\partial^2_t a}{a} + H^2 + \frac{k}{a^2}\right)\ . \tag{23}$$

3.2 Energy-momentum tensor for gravitational field

The corresponding stress-energy tensor of the gravitational field, $T^{\rm gr}_{\mu\nu} = -G_{\mu\nu}/8\pi G$, can be represented in terms of the energy density $\rho^{\rm gr}$ and the partial pressure $P^{\rm gr}$ of this gravitational component of the 'cosmological fluid':

$$T^{\rm gr}_{\mu\nu} = \rho^{\rm gr}u_\mu u_\nu + P^{\rm gr}(u_\mu u_\nu - g_{\mu\nu}) \tag{24}$$

$$= \frac{1}{8\pi G}\left(-3u_\mu u_\nu\left(H^2 + \frac{k}{a^2}\right) + (u_\mu u_\nu - g_{\mu\nu})\left(2\frac{\partial^2_t a}{a} + H^2 + \frac{k}{a^2}\right)\right)\ . \tag{25}$$

Here u^μ is the 4-velocity of the 'cosmological fluid'; in the comoving reference frame used it is $u^\mu = \delta^\mu_0$. We shall show that for all three Universes considered below, the energy-momentum tensor $T^{\rm gr}_{\mu\nu}$ makes sense, so that $\rho^{\rm gr}$ and $P^{\rm gr}$ do play a role, correspondingly, of the energy density stored in the gravitational field, and the partial pressure thermodynamically related to this energy. Later in Sec. 8 we shall discuss $T^{\rm gr}_{\mu\nu}$ for gravitational waves.

4 Einstein static Universe

4.1 Equation of state for gravitational field

For a static Einstein Universe, $\dot{a} = 0$, and from Eq.(25) one has

$$P^{\rm gr} = -\frac{1}{3}\rho^{\rm gr} = \frac{k}{8\pi G a^2} \ . \tag{26}$$

The equations (26) can be obtained from the pure phenomenology, i.e. without using and solving the Einstein equations. The magnitude of the energy density stored in the gravitational field component can be obtained directly from the Einstein action, since in the static case the energy density is the Lagrangian density with opposite sign:

$$\rho^{\rm gr} = -\mathcal{L} = \frac{1}{16\pi G}\mathcal{R} = -\frac{3k}{8\pi G a^2} \ . \tag{27}$$

Then the partial pressure of the gravitational field is obtained from the thermodynamic definition of pressure:

$$P^{\rm gr} = -\frac{d(\rho^{\rm gr} a^3)}{d(a^3)} = -\frac{1}{3}\rho^{\rm gr} \ . \tag{28}$$

Let us stress that this equation of state is applicable only for the gravitational field associated with the static Robertson-Walker metric.

4.2 Einstein solution from phenomenology

The Einstein static solution can be obtained without solving the Einstein equations by using the following phenomenological equations: (i) the gravineutrality condition, which states that the total gravitating energy density vanishes,

$$\rho^{\rm gr} + \rho^{\rm vac} + \rho^{\rm M} = 0 \ ; \tag{29}$$

(ii) the equilibrium conditions which states that the pressure of the system is zero,

$$P^{\rm gr} + P^{\rm vac} + P^{\rm M} = 0 \ ; \tag{30}$$

(iii) the equation of state for the vacuum component

$$P^{\rm vac} = -\rho^{\rm vac} = -\frac{\Lambda}{8\pi G} \ ; \tag{31}$$

(iv) the equation of state for the gravitational field component (28); and (v) the equation (27) for the energy density stored by the gravitational field. The latter two equations, (iv) and (v) were also obtained from phenomenology.

From (i-v) it follows that for the general equation of state for matter, the Einstein static Universe has the following induced cosmological constant and the curvature as functions of the matter fields:

$$\Lambda_{\rm Einstein} = 8\pi G\rho^{\rm vac} = 4\pi G(\rho^{\rm M} + 3P^{\rm M}) \ , \tag{32}$$

$$\rho^{\rm gr} = -\frac{3k}{8\pi G a^2} = -\frac{3}{2}(\rho^{\rm M} + P^{\rm M}) \ . \tag{33}$$

The equation (33) requires that, unless the matter is unconventional, the parameter k which characterizes the curvature of the 3D space must be positive ($k = +1$), i.e. the static Universe with conventional matter is closed.

5 de Sitter solution as a thermodynamic equilibrium state

We know that the static Einstein Universe is unstable. So let us turn to expanding Universes. Can one apply the same phenomenology of effective theory to the expanding or inflating Universes? Let show that the phenomenology does work in case of the dynamics of the flat Universe, i.e with $k = 0$:

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2 d\Omega^2\right) \ . \tag{34}$$

If $k = 0$ the Lagrangian (22) is quadratic in time derivative,

$$S^{\rm gr} = -\frac{1}{16\pi G}\int \sqrt{-g}\tilde{\mathcal{R}} \equiv -\frac{6}{16\pi G}\int a^3 \frac{(\partial_t a)^2}{a^2} \ . \tag{35}$$

That is why the energy density stored in the gravitational field equals the Lagrangian density:

$$\rho^{\rm gr} = -\frac{1}{16\pi G}\tilde{\mathcal{R}} = -\frac{3}{8\pi G}H^2 \ . \tag{36}$$

Since H is invariant under scale transformation $a \rightarrow \lambda a$, the equation of state for the gravitational field component,

$$P^{\rm gr} = -\rho^{\rm gr} \ , \tag{37}$$

is now the same as for the vacuum component. The equations (36) and (37), which we obtained here from phenomenology, reproduce equations (24) and (25) for the energy momentum tensor of the gravitational field.

The equilibrium state of the flat expanding Universe is now obtained from the (i) gravineutrality condition (29), and (ii) equilibrium condition (30), which together with (iii) the equation of state for the vacuum component, $P^{\rm vac} = -\rho^{\rm vac}$, and (iv) the equation of state for the gravitational field component, $P^{\rm gr} = -\rho^{\rm gr}$, read

$$\rho^{\rm gr} + \rho^{\rm vac} + \rho^{\rm M} = 0 \quad , \quad -\rho^{\rm gr} - \rho^{\rm vac} + P^{\rm M} = 0 \; . \tag{38}$$

Since $P^{\rm M} \neq -\rho^{\rm M}$, the only solution of the equations (38) is

$$\Lambda_{\rm de\ Sitter} = 8\pi G \rho^{\rm vac} = -8\pi G \rho^{\rm gr} = 3H^2 \quad , \quad P^{\rm M} = \rho^{\rm M} = 0 \; . \tag{39}$$

This is de Sitter Universe without matter, which is the steady state Universe because the Hubble parameter H is constant. In fact, the de Sitter Universe can be described in terms of the stationary but not static metric:

$$ds^2 = dt^2 - \frac{1}{c^2}\left(d\mathbf{r} - \mathbf{v}dt\right)^2 \; , \tag{40}$$

where the frame dragging velocity

$$\mathbf{v} = H\mathbf{r} \; . \tag{41}$$

6 Phenomenology of Gödel Universe

6.1 Rotating Universe

Let us now consider the different class of the Universes, the rotating Universe. In principle, it is not necessary to know the metric of this state, since it is enough to know, that it represents the local rotation of the vacuum with angular velocity Ω. However, for completeness we present the metric field obtained by Gödel as the solution of Einstein equations. In cylindrical coordinate system [13]

$$ds^2 = \left(dt + \Omega R^2 d\phi\right)^2 - \frac{dR^2}{1 + \frac{R^2}{R_0^2}} - R^2\left(1 - \frac{R^2}{R_0^2}\right) d\phi^2 - dz^2 \tag{42}$$

$$= dt^2 \frac{1+\frac{R^2}{R_0^2}}{1-\frac{R^2}{R_0^2}} - \frac{dR^2}{1+\frac{R^2}{R_0^2}} - R^2 \left(1-\frac{R^2}{R_0^2}\right) \left(d\phi + \frac{\Omega dt}{1-\frac{R^2}{R_0^2}}\right)^2 - dz^2 \ , \qquad (43)$$

$$R_0 = \frac{\sqrt{2}c}{\Omega} \ . \qquad (44)$$

When $R \ll R_0$, the metric corresponds to the Minkowski metric in the frame rotating with angular velocity Ω, that is why the vacuum in this Universe is locally rotating with velocity Ω. Thus the gravitational energy $\rho^{\rm gr}$ describes now the vacuum perturbation caused by rotation. In other words, the energy stored in the rotation, which is the excess of the energy with respect to the homogeneous vacuum state, represents the energy of the gravitational field component in the Gödel state.

As follows from the analogy with the vacuum state in condensed matter, if the rotation velocity is small, the energy of rotation can be written in general form as proportional to Ω^2. This means that the rotating vacuum must be characterized by the angular momentum (spin) $\mathbf{S}$, which is proportional to $\mathbf{\Omega}$. The coefficient χ in the linear response of the spin density $\mathbf{s}$ to the rotation velocity

$$\mathbf{s} = \chi \mathbf{\Omega} \ , \qquad (45)$$

plays the role of the spin susceptibility of the quantum vacuum.

The energy stored in the gravitational subsytem is thus

$$\rho^{\rm gr} = \frac{1}{2} \chi \Omega^2 \ . \qquad (46)$$

Note that the energy density of the gravitational field, which is stored in rotation, $\rho^{\rm gr} = (\chi/2)\Omega^2$, actually represents the effect of a local rotation. The global solid-body rotation corresponds to the Minkowski state in the rotating frame. In such a state the curvature $\mathcal{R} = 0$, and thus there is no energy associated with the gravitational field. The curvature in the solid-body rotating state is nullified due to the terms in $\mathcal{R}$, which are the full space derivatives. For the local rotations these counter-terms are absent, since they disappear after the integration by parts. This situation is similar to some 'paradoxes' related to the angular momentum in condensed matter.

Now we must find the spin susceptibility of the vacuum, χ, which characterizes the local response of the vacuum spin density to the rotation.

6.2 Spin susceptibility of the vacuum

The spin susceptibility of the vacuum is

$$\chi = -\frac{1}{4\pi G\sqrt{-g}} \ . \tag{47}$$

This diagravimagnetic response of the vacuum to rotation follows from the gravitational interaction of two spins in the vacuum. If two bodies at points $\mathbf{r}$ and $\mathbf{r}'$ have spins $\mathbf{S}$ and $\mathbf{S}'$ respectively, their interaction energy in the post-Newtonian approximation is [14]

$$U^{\rm gr} = -\mathbf{S}\cdot\boldsymbol{\Omega}(\mathbf{r}) = -\mathbf{S}'\cdot\boldsymbol{\Omega}(\mathbf{r}') = -\frac{G}{\sqrt{-g}}\,\frac{\mathbf{S}\cdot\mathbf{S}' - 3(\mathbf{n}\cdot\mathbf{S})(\mathbf{n}\cdot\mathbf{S}')}{|\mathbf{r}-\mathbf{r}'|^3} \ , \tag{48}$$
$$\mathbf{n} = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} \ , \ \sqrt{-g} = \frac{1}{c^3} \ .$$

This interaction via the vacuum demonstrates that the spin susceptibility of the vacuum is given by Eq.(47). The sign of the gravitational interaction energy of spins in Eq.(48) is opposite to that of the magnetic dipole-dipole interaction of their magnetic moments. For example, the magnetic dipole-dipole interaction of the electron spins is

$$U^{\rm magn} = \frac{\hbar\alpha}{M^2\sqrt{-g}}\,\frac{\mathbf{S}\cdot\mathbf{S}' - 3(\mathbf{n}\cdot\mathbf{S})(\mathbf{n}\cdot\mathbf{S}')}{|\mathbf{r}-\mathbf{r}'|^3} \ , \tag{49}$$

where α is the fine structure constant, and M is the rest energy of the electron.

The Eq.(47) can be also obtained from the frame dragging effect caused by the spinning body. In the post-Newtonian approximation, the frame dragging velocity $\mathbf{v}$ (with $\nabla\times\mathbf{v} = 2\boldsymbol{\Omega}$) caused by a body at $\mathbf{r}=0$ with the spin $\mathbf{S}^0$ is determined by the variation of the following energy:

$$\frac{1}{2}\chi\int\sqrt{-g}\boldsymbol{\Omega}^2 - \mathbf{S}^0\cdot\boldsymbol{\Omega}(\mathbf{r}=0) \ . \tag{50}$$

The variation over $\mathbf{v}$ gives the following frame dragging field around the body:

$$\mathbf{v} = \frac{1}{2\pi\chi\sqrt{-g}}\frac{\mathbf{S}^0\times\mathbf{r}}{r^3} \ . \tag{51}$$

Comparing this with Eq.(9.5.18) of the book [15]

$$\mathbf{v} = -2G\frac{\mathbf{S}^0 \times \mathbf{r}}{r^3} , \tag{52}$$

one obtains Eq.(47) for the spin susceptibility of the vacuum.

6.3 Equation of state for the gravitational field and equilibrium state

Since the spin is conserved quantity, the relevant gravitational energy which must be used to obtain the partial pressure of the gravitational subsystem, must be expressed in terms of the total spin $\mathbf{S} = V\mathbf{s}$, where V is the volume of the system:

$$E^{\rm gr} = \rho^{\rm gr} V = \frac{\mathbf{S}^2}{2\chi V} . \tag{53}$$

The equation of state for the gravitational subsystem is obtained from Eq.(53) by varying over the volume at fixed total momentum $\mathbf{S}$:

$$P^{\rm gr} = -\frac{dE^{\rm gr}}{dV} = \rho^{\rm gr} . \tag{54}$$

Then from the phenomenological equations (29–31) and (54) one obtains the following response of the cosmological cosntant to the matter field in the rotating Universe:

$$\Lambda_{\rm Goedel} = 8\pi G \rho^{\rm vac} = -4\pi G(\rho^{\rm M} - P^{\rm M}) , \tag{55}$$

$$\rho^{\rm gr} = -\frac{1}{2}(\rho^{\rm M} + P^{\rm M}) . \tag{56}$$

For the cold matter, $P^{\rm M} = 0$, the induced cosmological constant is

$$\Lambda_{\rm Goedel} = 8\pi G \rho^{\rm vac} = -\Omega^2 = -4\pi G \rho^{\rm M} , \tag{57}$$

where we used Eq.(46): $\rho^{\rm gr} = \chi\Omega^2/2 = -\Omega^2/8\pi G$. The equation (57) has been obtained by Gödel from the solution of the Einstein equations [13].

7 Modification of Einstein equation and relaxation of the vacuum energy

7.1 Cosmological constant as evolving parameter

Till now we have not got any new information, since all these well known results were originally obtained by solving the Einstein equations. So, what is the point in the rederivation of the old results? This served to demonstrate that actually there are no cosmological constant puzzles. The cosmological constant is not an absolute constant: we have seen that the gravitating vacuum energy is adjusted to different types of the perturbations of the vacuum in addition to the energy density of matter: (i) to the curvature of space in the Einstein closed Universe; (ii) to the expansion rate in the de Sitter Universe; and (iii) to the rotation in the Gödel Universe. In all these cases the gravitating vacuum energy is zero in the absence of perturbations, being proportional to the energy density of perturbations. This is in a full agreement with the Gibbs-Duhem relation applicable to any quantum vacuum, and shows that (i) the cosmological constant is not huge, since according to the Gibbs-Duhem relation the contribution of zero point fluctuations to the vacuum energy is cancelled by the trans-Planckian degrees of freedom; (ii) the cosmological constant is non-zero, since the perturbations of the vacuum state induce the vacuum energy; and (iii) the gravitating vacuum energy is on the order of the energy density of matter and/or of other perturbations.

There are other consequences of this phenomenological approach. For example, the Gibbs-Duhem relation does not discriminate between the false vacuum and true vacuum. The only requirement is that the vacuum state must correspond to a local minimum or a saddle point of the energy functional. This means that (iv) the false vacuum is also non-gravitating if it is not perturbed. This leads to a rather paradoxical conclusion: (v) if the cosmological pahse transition from the false to true vacuum occurs at low temperature, the cosmological constant is (almost) zero above the cosmological phase transition, but below the transition it will also become zero after some transient period. This transient period can be accompanied by the inflationary stage of the expanding Universe.

Since the cosmological constant is not a constant, but an evolving parameter, which is ajusted to the vacuum perturbations, the remaining cos-

mological constant problem is (vi) to understand how it evolves in time. The Einstein equation does not allow us to obtain the time dependence of the cosmological constant. This is because of the Bianchi identities, $G^{\nu}_{\mu;\nu} = 0$, and the covariant conservation law for matter fields (quasiparticles), $T^{\nu\mathrm{M}}_{\mu;\nu} = 0$, which together lead to $\partial_\mu \Lambda = 0$. To describe the evolution of the cosmological constant we must modify the Einstein equations to allow Λ to relax to its equilibrium value. Thus the relaxation term must be added which violates the general covariance. This correction comes from the trans-Planckian physics, and thus it must contain the Planck energy scale. The Planck physics can also violate the Lorentz invariance: the dissipation implies the existence of the preferred reference frame, which is the natural ingredient of the trans-Planckian physics.

7.2 Dissipation in Einstein equation

The dissipation in the Einstein equation can be introduced in the same way as in two-fluid hydrodynamics [16] which serves as the non-relativistic analog of the self-consistent treatment of the dynamics of the vacuum component (the superfluid component of the liquid) and the matter component (the normal component of the liquid) [5]. We must add the dissipative part $T^{\mathrm{diss}}_{\mu\nu}$ to the total energy-momentum tensor in Eq.(9):

$$T^{\mathrm{M}}_{\mu\nu} + T^{\mathrm{vac}}_{\mu\nu} + T^{\mathrm{gr}}_{\mu\nu} + T^{\mathrm{diss}}_{\mu\nu} = 0 \; . \tag{58}$$

In contrast to the conventional dissipation of the matter, such as viscosity and thermal conductivity, this term is not the part of $T^{\mathrm{M}}_{\mu\nu}$. It describes the dissipative back reaction of the vacuum, which does not influence the matter conservation law $T^{\nu\mathrm{M}}_{\mu;\nu} = 0$. The condensed-matter example of such relaxation of the variables describing the fermionic vacuum is provided by the dynamic equation for the order parameter in superconductors – the time-dependent Ginzburg-Landau equation which contains the relaxation term (see e.g. the book [17]).

In the lowest order of the gradient expansion, the dissipative part $T^{\mathrm{diss}}_{\mu\nu}$ of the stress tensor describing the relaxation of Λ must be proportional to the first time derivative of Λ. Since $T^{\mathrm{diss}}_{\mu\nu}$ is a tensor, the general description of the vacuum relaxation requires introduction of several relaxation times. In the isotropic space we have only two such relaxation parameters, in the

energy and pressure sectors:

$$T^{\rm diss}_{\mu\nu} = \left(\tau_1 u_\mu u_\nu + \tau_2 (g_{\mu\nu} - u_\mu u_\nu)\right) \partial_t \Lambda \ . \tag{59}$$

Here the 4-velocity u_μ selects the reference frame of the trans-Planckian physics, which may or may not coincide with the comoving reference frame. In principle, the two reference frames can be dynamically coupled, for example there can be a mutual dissipative friction which forces the two frames to be aligned in equilibrium. This would be analogous to the Gorter-Mellink [18] mutual friction force between different components of a superfluid liquid in condensed matter, and it could also serve as a source of dissipation.

The preferred reference frame is an important issue in the effective gravity. As the condensed matter analogy suggests, [5] there can be several reference frames: (i) the reference frame of the absolute spacetime; (ii) the preferred reference frame, in which the non-covariant corrections to the effective action coming from the Planck-scale physics have the most simple form; (iii) the comoving frame; (iv) the frame of the matter; and finally (v) the so-called frame of texture, which corresponds to the preferred topological frame discussed in [19].

Further we assume that the system is close to equilibrium, so that u_μ in Eq.(59) is the 4-velocity of the comoving reference frame. Then the next problem is to find the relaxation parameters, τ_1 and τ_2, which can be the functions of the matter fields. However, in principle, these functions can be treated as phenomenological, which can be extracted from the observations.

Let us consider several the most simple examples of relaxation of the cosmological constant.

7.3 Cosmological constant as integration constant

Let us start with the most simple case when $\tau_1 = \tau_2 = \text{constant} \equiv \tau_\Lambda$. In this case the dissipative part of the stress tensor is proportional to the metric tensor, $T^{\rm diss}_{\mu\nu} = \tau_\Lambda g_{\mu\nu} \partial_t \Lambda$. The Bianchi identities require that $\partial_t (\Lambda + \tau_\Lambda \dot{\Lambda}) = 0$, which gives $\Lambda + \tau_\Lambda \partial_t \Lambda = \Lambda_0$. From equations (59) and (8) it follows that $T^{\rm vac}_{\mu\nu} + T^{\rm diss}_{\mu\nu} = \frac{\Lambda_0}{8\pi G} g_{\mu\nu}$, i.e. the integration constant Λ_0 plays the role of the cosmological constant. The other examples when the cosmological constant arises as an integration constant are well known in the literature (see reviews [1, 4]). In our example, the vacuum energy density is not a constant, but

exponentially relaxes to Λ_0:

$$\Lambda(t) = \Lambda_0 + \Lambda_1 \exp\left(-\frac{t}{\tau_\Lambda}\right) \ , \tag{60}$$

where Λ_1 is another integration constant. But experimentally we cannot resolve such evolution of the vacuum energy, unless this evolution influences the equations for the matter fields. That is why the dissipative part of the stress tensor should not be proportional to the metric tensor, and we really need $\tau_2 \neq \tau_1$.

7.4 Flat Universe with two relaxation parameters

In such a general case the dynamics of the cosmological constant is determined by the Einstein equations modified by the relaxation term (59). Let us consider this for a flat Robertson-Walker Universe. For simplicity, we assume that the Planck-scale reference frame is aligned with the comoving frame. In this case one obtains the following equations

$$-3H^2 + \Lambda + \tau_1 \dot{\Lambda} + 8\pi G \rho^{\rm M} = 0 \ , \tag{61}$$

$$3H^2 + 2\dot{H} - \Lambda - \tau_2 \dot{\Lambda} + 8\pi G p^{\rm M} = 0 \ . \tag{62}$$

Because of the presence of the non-covariant relaxation term, the covariant conservation law for matter does not follow now from the Bianchi identities. That is why the above two equations must be supplemented by the covariant conservation law to prevent the creation of matter:

$$a\frac{\partial}{\partial a}(\rho^{\rm M} a^3) = p^{\rm M} a^3 \ . \tag{63}$$

Together with the equation of state for matter, these give four equations for the four functions H, Λ, $\rho^{\rm M}$ and $P^{\rm M}$.

Let us discuss some consequences of these equations.

7.5 Relaxation after cosmological phase transition

Let us start with the case when the relaxation occurs only in the pressure sector, i.e. $\tau_1 = 0$, and assume also that the ordinary matter is cold, i.e. its pressure $p^{\rm M} = 0$, which gives $\rho^{\rm M} \propto a^{-3}$. Then one finds two classes of

solutions: (i) $\Lambda =$ constant; and (ii) $H = 1/(3\tau_2)$. The first one corresponds to the conventional expansion with the constant Λ-term and the cold matter, so let us discuss the second solution, $H = 1/(3\tau_2)$.

In the case when $\tau_2 =$ constant, one finds that the Λ-term and the energy density of matter $\rho^{\rm M}$ exponentially relaxe to $1/(3\tau_2^2)$ and to 0 respectively:

$$H = \frac{1}{3\tau_2},\ \Lambda(t) = \frac{1}{3\tau_2^2} - 8\pi G\rho^{\rm M}(t),\ \frac{\rho^{\rm M}(t)}{\rho^{\rm M}(0)} = \exp\left(-\frac{t}{\tau_2}\right). \qquad (64)$$

Such solution describes the behavior after the cosmological phase transition.

The cosmological phase transitions also impose the fine-tuning problem for the cosmological constant. If, for example, the electroweak phase transition occurs according to the Standard Model of the electorweak interactions (see e.g. [20]), then after this phase transition the vacuum energy is reduced by the value of the energy of the Higgs field. This is about 10^{50} larger than the observational upper limit for the vacuum energy. It means that the primordial value of the vacuum energy (i.e. before the phase transition) must be fine-tuned to cancel 50 decimal.

Let us consider how the same problem is resolved in condensed matter on the exampe of the phase transition which occurs at $T = 0$. The proper example is the first-order transition between two superfluid vacua, A and B, in the ^{3}He liquid at $T = 0$. According to the Gibbs-Duhem relation, which is applicable to the false vacuum too, the relevant vacuum energy before the transition, i.e. that of the false vacuum, is $E^{\rm false\ vac} = E - \mu^{\rm false\ vac}N = 0$. Immedieately after the phase transition to the true vacuum, the vacuum energy acquires the big negative value thus violating the Gibbs-Duhem relation. However, after that the chemical potential $\mu^{\rm false\ vac}$ starts to relax to the new value $\mu^{\rm true\ vac}$ to restore the Gibbs-Duhem relation in a new equilibrium. As a result, after some time the relevant energy of the true vacuum also becomes zero: $E^{\rm true\ vac} = E - \mu^{\rm true\ vac}N = 0$.

Let us extend this scenario to the cosmological vacuum and the cosmological phase transition. If this analogy is correct, this implies that if the temperature is low, the cosmological 'constant' Λ is (almost) zero before the transition. Immedieately after the transition it drops to the negative value; but after some transient period it relaxes back to zero. The equation (64) just corresponds to the latter stage. But this solution demostrates that in its relaxation after the phase transition, the Λ-term crosses zero and finally

becomes a small positive constant determined by the relaxation parameter $1/\tau_2$ which governs the exponential de Sitter expansion.

7.6 Dark energy as dark matter

Let us now allow τ_2 to vary. In condensed matter the relaxation and dissipation are determined by quasiparticles, which play the role of matter. According to analogy, the relaxation term must be also determined by matter. The inverse relaxation time must contain the Planck scale $E_{\rm Planck}$ in the denominator, since the relaxation of Λ must disappear in the limit of infinite Planck energy, when the general covariance is restored. The lowest-order term, which contains the $E_{\rm Planck}$ in the denominator, is $\hbar/\tau_2 \sim T^2/E_{\rm Planck}$, where T is the characteristic temperature or energy of matter. In case of radiation it can be written in terms of the radiation density:

$$\frac{1}{3\tau_2^2} = 8\pi\alpha G\rho^{\rm M} \ , \tag{65}$$

where α is the dimensionless parameter. If Eq.(65) can be applied to the cold baryonic matter too, then the solution of the class (ii) becomes again $H = 1/(3\tau_2)$, but now τ_2 depends on the matter field. This solution gives the standard power law for the expansion of the cold flat universe and the relation between Λ and the baryonic matter $\rho^{\rm M}$:

$$a \propto t^{2/3},\ 8\pi G\rho^{\rm M} = \frac{4}{3\alpha t^2},\ H = \frac{2}{3t},\ \Lambda = (\alpha - 1)8\pi\ G\rho^{\rm M}. \tag{66}$$

This solution is completely equivalent to the flat expanding Universe without the cosmological constant, which follows from the Einstein equations with the cold matter at critical density. The reason is that the effective vacuum pressure in Eq.(62), which comes from the vacuum and the dissipative Λ-terms, cancel each other, $p_\Lambda = -(\Lambda + \tau_2\dot{\Lambda})/8\pi G = 0$. This means that in this solution the vacuum behaves as the cold dark matter. Altogether, the energy density of this cold dark matter and that of the ordinary matter form the critical density corresponding to the flat universe in the absence of the vacuum energy:

$$\rho^{\rm vac} + \rho^{\rm M} = \rho_c = \frac{1}{6\pi G t^2} \ . \tag{67}$$

This demonstrates that in some cases the vacuum can serve as the origin of the non-baryonic dark matter.

7.7 Analog of quintessence

These examples are too simple to describe the real evolution of the present universe and are actually excluded by observations [2]. The general consideration with two relaxation functions is needed. In this general case, the effective equation of state which comes from the reversible and dissipative Λ-terms corresponds to the varying in time parameter w_Q

$$w_Q(t) = \frac{p_\Lambda}{\rho_\Lambda} = -\frac{\Lambda + \tau_2\dot{\Lambda}}{\Lambda + \tau_1\dot{\Lambda}} \ . \tag{68}$$

Such an equation of state is usually ascribed to the quintessence, a kind of the scalar field, which provides the varying negative pressure. The recent observational bounds on w_Q can be found, for example, in Refs. [21, 22].

8 On energy and momentum of gravitational waves

Above we considered three steady-state Universes and found that the energy-momentum tensor of gravitational field determined as $T^{\rm gr}_{\mu\nu}$ in Eq.(10) makes sense, so that $\rho^{\rm gr}$ and $P^{\rm gr}$ do play a role, correspondingly, of the energy density stored in the gravitational field, and the partial pressure thermodynamically related to this energy. Here we apply this definition (10) to the gravitational waves.

In the absence of matter and cosmological cosntant the Einstein equations read as $T^{\rm gr}_{\mu\nu} = 0$, which means that the gravitational field in empty space has zero energy and momentum. At first glance this leads to the paradoxical conclusion that the gravitational wave does not carry energy and momentum.

The origin of this paradox is in the non-linear nature of the Einstein equations, which encodes the self interaction of the gravitational field in general relativity. Let us consider the perturbation theory describing the propagation of waves of small amplitude, $g_{\mu\nu} = \eta_{\mu\nu} + h^{(1)}_{\mu\nu} + h^{(2)}_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski flat metric and $h_{\mu\nu} \ll \eta_{\mu\nu}$ is the perturbation. In linear approximation, the equation $T^{{\rm gr}(1)}_{\mu\nu} = 0$ represents the wave equation for the function $h^{(1)}_{\mu\nu}$ describing gravitons with two polarizations $h^{(1)}_{12}$ and $h^{(1)}_{22} = -h^{(1)}_{11}$ propagating, say, along the z-axis.

Now let us go to the second-order terms. There is the contribution to $T^{\rm gr(2)}_{\mu\nu}$ which is quadriatic in $h^{(1)}_{\mu\nu}$. It is non-zero and it represents the energy and momentum of the graviton (see [15] Sec. 10.3):

$$T^{\rm graviton}_{\mu\nu} = \frac{k_\mu k_\nu}{8\pi G}\ \left(|h_{12\mathbf{k}}]^2 + \frac{1}{4}|h_{22\mathbf{k}} - h_{11\mathbf{k}}]^2\right)\ , \tag{69}$$

where $h_{\mathbf{k}}$ are the ampltudes of the propagating waves. The equation $T^{\rm gr(2)}_{\mu\nu} = 0$ in this quadratic approximation is restored by the second-order correction $h^{(2)}_{\mu\nu}$. It describes the response of the gravitational field to the energy and momentum of the graviton, which serve as a source of the additional gravitational field (see [15] Sec. 7.6):

$$T^{\rm graviton}_{\mu\nu} + T^{\rm gravity\ field}_{\mu\nu} = 0\ ,\ T^{\rm gravity\ field}_{\mu\nu} = -\frac{1}{8\pi G}G^{(2)}_{\mu\nu}\ . \tag{70}$$

Here $G^{(2)}_{\mu\nu}$ is the Einstein tensor of the gravitational field $h^{(2)}_{\mu\nu}$ induced by the gravitating graviton. It depends linearly on $h^{(2)}_{\mu\nu}$:

$$\begin{aligned} 2G^{(2)}_{\mu\nu} = \partial^\alpha\partial_\alpha h^{(2)}_{\mu\nu} - \partial^\alpha\partial_\mu h^{(2)}_{\nu\alpha} - \partial^\alpha\partial_\nu h^{(2)}_{\mu\alpha} + \partial_\mu\partial_\nu h^{\alpha(2)}_{\alpha} \\ -\eta_{\mu\nu}\left(\partial^\alpha\partial_\alpha h^{\beta(2)}_{\beta} - \partial^\alpha\partial^\beta h^{(2)}_{\alpha\beta}\right). \end{aligned} \tag{71}$$

Thus, in spite of the fact that the energy-momentum tensor of the gravitational field is zero in empty space, $T^{\rm gr}_{\mu\nu} = 0$, it can be decomposed in two terms which cancel each other: the energy-momentum tensor of the graviton $T^{\rm graviton}_{\mu\nu}$ and that of the gravitational field induced by the energy and momentum of the graviton, $T^{\rm gravity\ field}_{\mu\nu}$. Of course, such separation is not covariant, but any separation between different degrees of freedom, i.e. between different subsystems, does not necessarily obey the symmetry of the whole system. The mere presence of a graviton violates the general covariance, since it introduces the distinguished reference frame. The graviton plays the same role as any other matter which gives rise to the preferred reference frame comoving with matter. However, the energy-momentum tensor of the whole system, $T^{\rm gr}_{\mu\nu}$, obeys both the true conservation law and covariant conservation law, simply because it is zero, and thus there is no contradiction between the general covariance and the conservation of energy and momentum.

There is a deep difference between the covariant energy-momentum tensor $T^{\rm gr}_{\mu\nu}$ and the non-covariant energy-momentum tensor of the graviton $T^{\rm graviton}_{\mu\nu}$.

The first one is obtained as the functional derivative of the action with respect to the the whole metric, while the second one is obtained by variation over the metric which serves as the background metric for graviton, i.e. from which some part is excluded – the oscillating part involved in the graviton.

A similar situation occurs in hydrodynamics, which also represents the non-linear theory. The propagation of sound wave in liquids is governed by the dynamical acoustic metric provided by the density ρ and velocity $\mathbf{v}$ fields of the moving liquid. These fields play the role of the gravity field in general relativity, while the sound waves – propagating perturbations $\rho^{(1)}$ and $\mathbf{v}^{(1)}$ – play the role of gravitational waves [23, 5]. Here again from the same initial field, now the hydrodynamic fields ρ and $\mathbf{v}$, two subsystems are formed: (i) the phonon and (ii) the smooth acoustic metric, which governs the phonon propagation and is influenced by the phonon due to back reaction. Thus there are also two metrics at our disposal when we discuss the momentum of the sound wave: the metric of the fundamental spacetime in which the liquid lives, and the acoustic metric of the effective spacetime in which the phonon lives [23]. The 'real' energy and momentum are obtained by differentiating with respect to the 'real' (fundamental) metric, while those obtained by differentiation with respect to the acoustic (effective) metric are called the pseudoenergy and pseudomomentum; many paradoxes in condensed matter physics arise if these two notions are not discriminated (see references in [23]).

The 'real' momentum density of the liquid associated with the sound wave (phonon) in Eq.(13.52) of [23] consists of two parts:

$$\mathbf{p}^{\rm liquid} = \langle \rho^{(1)} \mathbf{v}^{(1)} \rangle + \mathbf{v} \langle \rho^{(2)} \rangle \ . \tag{72}$$

The first term in Eq.(72) is the pseudomomentum of the phonon (the analog of graviton), while the second term represents the momentum coming from the back reaction of the liquid (analog of the gravitational field produced by the gravitating graviton). Here $\langle \rho^{(2)} \rangle$ is the second-order perturbation of the density of the liquid caused by the phonon, which is the analog of $h^{(2)}_{\mu\nu}$.

Because of the momentum conservation law, which comes from the translational symmetry of the whole system, the total momentum of the liquid must be always zero, $\mathbf{p}^{\rm liquid} = 0$. However, each of the two momenta, of the phonon and of the liquid, is non-zero and can be measured separately. The mere presence of a phonon in the liquid violates the translational invariance, and thus the pseudomomentum of the phonon is non-zero.

Condensed matter systems provide many examples, when the energy-momentum tensor, though is well defined on the microscopic level, cannot be localized when it is written in terms of the limited number of the infrared collective variables (see e.g. [24] for ferromagnets). One can push further the analogy between the quantum vacuum in general relativity and the quantum vacuum in condensed matter. One can imagine that the metric $g_{\mu\nu}$ arises only in the low-energy corner while the whole vacuum lives in the fundamental spacetime, Lorentzian or Galilean, appropriate for the microscopic (trans-Planckian) physics [5]. If it is so, then even the covariant energy-momentum tensor $T^{\rm gr}_{\mu\nu}$, which is expressed in terms of the collective field $g_{\mu\nu}$, does not describe the real energy and momentum of the gravitational field. The latter remain unknown and will be revealed only at high energy, where the non-covariant corrections caused by the Planck physics will indicate the preferred reference frame of the quantum vacuum.

9 Discussion

If the gravity does belong to the class of the emergent phenomena [25], we can obtain some useful consequences of that. The main message is that in the effective gravity the equilibrium time-independent vacuum state without matter is non-gravitating, i.e. its relevant vacuum energy, which is responsible for gravity, is zero. On the other hand, if the vacuum is perturbed, the cosmological constant is non-zero, and it is ajusted to the perturbations. If the case of the steady state perturbations, the response of the cosmological constant can be found from the Einstein equations. It appears, however, that in some of these cases it is not necessary to solve Einstein equations, to obtain this response. The response to the curvature, steady state expansion and rotation can be obtained from the purely phenomenological approach, as we discussed in this paper. The response of the vacuum energy to matter in the world without gravity (i.e. when the Newton constant $G = 0$) was discussed in [26].

If the general case of the time-dependent perturbations, the cosmological constant is an evolving parameter rather than the constant. The process of relaxation of the cosmological constant, when the vacuum is disturbed and out of the thermal equilibrium, requires some modification of the Einstein equation, since the Bianchi identities must be violated to allow the cosmo-

logical constant to vary. In contrast to the phenomenon of nullification of the cosmological constant in the equilibrium vacuum, which is the general property of any quantum vacuum and does not depend on its structure and on details of the trans-Planckian physics, the deviations from the general relativity can occur in many different ways, since there are many routes from the low-energy effective theory to the high-energy 'microscopic' theory of the quantum vacuum. However, it seems reasonable that such modification can be written in the general phenomenological way, as for example the dissipative terms are introduced in the hydrodynamic theory. Here we suggested to describe the evolution of the Λ-term by two phenomenological parameters (or functions) – the relaxation times, and demonstrated that this term ic responsible for the quintessence.

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